2001. Consider the cards one by one: use a conditioning as opposed to a combinatorics argument.

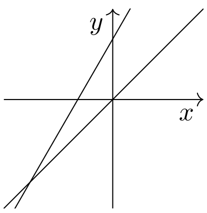
2002. In a proper algebraic fraction, the degree of the polynomial in the numerator is strictly lower than the degree of the polynomial in the denominator.

2003. (a) There is a single word in the question which implies that the accelerations of the 2 and 4 kg masses are the same.

   (b) Draw force diagrams for each of the three blocks separately, with accelerations a_1 for the 2 and 4 kg masses and a_2 for the 3 kg mass. Find F_{max} at each surface. Then solve the three equations of motion, assuming maximal friction.

   Remember that, if maximal friction produces a negative acceleration, then, in fact, friction is not maximal: the object doesn't move.

2004. Find the angle each line makes with the x axis. Then chase these angles into the triangle formed by the lines and the x axis.



2005. (a) Consider $\text{lcm}(4, 6)$.

   (b) Find the periods of the terms $\text{F}(2x)$ and $\text{G}(3x)$, by considering input transformations, viz. stretches in the x direction.

2006. Using $y - y_1 = m(x - x_1)$, set up a general equation for any straight line through $(2, -2)$. The gradient m will remain an unknown. Then use Δ to ensure that such a line has precisely one intersection with the parabola.

——— Alternative Method ———

Find the equation of the tangent line at $(a, a^2 - a)$, using calculus. Substitute the point $(2, -2)$ into this, and solve for a.

2007. Write the sum out longhand, and solve.

2008. Consider rolling the m-sided die first.

2009. Use $\sin^2\theta + \cos^2\theta \equiv 1$ to find the value of $\cos^2 36°$. Take the positive square root and reciprocate to find an (unsimplified) expression for $\sec 36°$. Then rationalise the denominator using its conjugate.

2010. Consider the sign of the *second* derivative. Use it to show that the *first* is increasing everywhere.

2011. Rearrange and square both sides, but remember that squaring may (in this case, it does) introduce new solutions.

2012. In each case, sketch the graphs.

2013. Since f is linear, the signed areas between $y = \text{f}(x)$ and the x axis form trapezia/triangles. Use this to work out the values of $\text{f}(1/2)$ and $\text{f}(3/2)$.

2014. Draw a force diagram for one bauble, and resolve vertically. Note that, due to symmetry, there can be no vertical contact forces between the baubles.

2015. Use the scale factor $\frac{\pi}{180}$ to convert input angles in degrees to angles in radians. Then substitute $\frac{\pi}{180}\theta$ into $\cos\theta \approx 1 - \frac{1}{2}\theta^2$ and simplify.

2016. Call the initial speed u, and generate vertical and horizontal equations. These are simultaneous equations in u and t. Substitute the horizontal into the vertical, eliminating t.

2017. It's easiest to ignore the $\frac{1}{9}$. Use 3D Pythagoras to equate the magnitude of the vector in the bracket to 9. Show that this equation has no real roots.

——— Alternative Method ———

Set q to zero and find the magnitude.

2018. Use the discriminant.

2019. Consider the unit circle.

2020. To be invertible, a function must be one-to-one over its domain. Consider a positive quartic. As $x \to \pm\infty$, $\text{f}(x) \to \infty$. So, $y = \text{f}(x)$ must have a turning point.

2021. (a) Reflection in its line of symmetry leaves the normal distribution unchanged.

   (b) Consider (a), plus a translation.

   (c) Consider this as a translation and a stretch. Remember that variance is a squared measure of spread.

2022. Find the correct move for ×, and then explain why ○ cannot respond.

2023. Consider the bottom two equations together, with the standard theorem concerning products equal to zero.

2024. This is the reverse chain rule.

2025. Draw the possibility space as a 6×6 grid. On it, shade the restricted possibility space, and tick the successful outcomes. Then use $p = \frac{\text{successful}}{\text{total}}$.

2026. (a) Consider the NIII pair of the frictional force on the ground. This force acts on the truck.

    (b) Set up NII for the whole system to work out the acceleration. Then set up NII for the trailer to find the tension.

    (c) Draw a new set of force diagrams, with a new braking force B. Use the equation of motion for the truck to find the tension. You don't need to find B.

2027. Multiply by x^2: this is a quadratic in x^3.

2028. Consider the value of $\cos\theta$ for small angles.

2029. Put the fractions over a common denominator.

2030. In (b) and (c), this is implicit differentiation. Treat y as a function of x, and use the chain rule.

2031. The set is $(2, \infty)$, not $[2, \infty)$.

2032. This boils down to showing that the range of $\frac{1+a}{1-a}$ is $\mathbb{R} \setminus \{k\}$, for some value k. Write $\frac{1+a}{1-a}$ as a constant plus a proper algebraic fraction.

2033. Name the couples A, B, C. Person A_1 can be placed arbitrarily. Then, find the probability that person A_2 sits in an adjacent seat. Then consider the possible orders of $B_1B_2C_1C_2$.

2034. First, solve the equation $f'(2) = 0$; there are two roots, one of which can be ruled out immediately using the point $(2, 32)$. This gives the value of q. Substitute back in to find the value of p.

2035. This is a quadratic in x^{10}. Find the prime factors of 5120 and you'll be able to factorise the quadratic directly. Or else there's nothing wrong with a bit of brute force: use the quadratic formula to give $x^{10} = ...$

2036. There are many possible scenarios: lift, ceiling, bungee cord, spaceship...

2037. It doesn't matter what interval is picked, nor how the values are distributed. The only thing that matters is that the four values x_1, x_2, x_3, x_4 have the *same* distribution.

2038. Draw a good sketch of the scenario, and then use a vector method.

2039. Write b as $a^{\log_a b}$, and consider the transformation as a replacement of x by $(\log_a b)x$.

2040. The median is a measure of central tendency and the IQR is a measure of spread.

2041. For *increasing*: show that the first derivative is positive for $x \in (0, \infty)$. For *concave*: show that the second derivative is negative for $x \in (0, \infty)$.

2042. The language ends up meaning "Show that, at the marked vertex, the two dotted lines trisect the right angle."

2043. Using the chain rule, find the derivative f'. Then substitute this in; you'll need to use a Pythagorean trig identity to simplify.

2044. (a) Remember to draw a force diagram *for* the girl, i.e. drawing forces *on* the girl. If she exerts a force downwards and backwards, then the ground exerts a force upwards and forwards.

    (b) Use *suvat*.

    (c) And again!

2045. This is a standard result used in simplifying the standard deviation/variance formulae. Prove it by expanding the brackets, then splitting the sum up, then using the definition of the sample mean $\bar{x}$.

2046. (a) Substitute the equation of the line into the equation of the curve.

    (b) Show that there is a point of intersection which has a repeated root.

2047. The symbol in between the two sets $A \setminus B$ is "set minus". So, consider A and B separately, before removing anything that lies in B from A.

2048. This is not as complicated as it sounds. Translate the first sentence into algebra, before integrating twice. Then consider the relevant equation.

2049. Use index laws to manipulate the expressions.

2050. Use the product rule twice.

2051. Draw a Venn diagram. Restrict the possibility space to B, and show that, with B given, A is more likely to occur than not.

2052. Find the derivative using the chain rule, and show that it is positive for all x.

2053. A counterexample is simply any example which counters, and hence disproves, a claim. So, list the first few primes and look for an AP.

2054. The first and last statements are true, while the second is not.

2055. Factorise the difference of two squares.

— Alternative Method —

Rearrange and take the square root.

2056. Count the number of diagonals which emerge from any one vertex, then multiply by the number of vertices. This will overcount by a factor of two.

2057. At a point of inflection, the second derivative is zero and changes sign. Since you're looking for a *greatest* number of points of inflection, you don't need to worry about the sign change. Consider the type of equation generated by setting the second derivative to zero.

2058. Put the integrand over a common denominator. Then use the reverse chain rule.

2059. Choose one vertex, without loss of generality. Then consider the number of ways of choosing two from the remaining five vertices.

2060. An iteration $x_{n+1} = \mathrm{f}(x_n)$ has a fixed point where the equation $x = \mathrm{f}(x)$ is satisfied.

2061. (a) Set $t = 0$ in the rate formula.
(b) Calculate the rate at $t = 0$ and $t = 280$.
(c) Total methane production V is given by the definite integral of the rate $\frac{dV}{dt}$ of production, between $t = 0$ and $t = 280$. Compare this to the amount produced with a constant rate as in part (a).

2062. "As a polynomial in $(3x+2)$" means, in this case, as a quadratic in $(3x+2)$, i.e. as

$$a(3x+2)^2 + b(3x+2) + c.$$

2063. Write $xy = k$ and make y the subject. Then find the derivatives. The factors of k should cancel.

2064. Distribute the operation "taking the mean" over the two terms in the difference, i.e. find the mean of the first term and the mean of the second term, and subtract these. Then use the definition of the variance to get an expression for Σx_i^2.

2065. Draw a force diagram, using R_1 and R_2 for the two reaction forces. Then set up two equations: vertical equilibrium and moments about e.g. the left-hand support. Solve for e.g. R_2 and then R_1.

2066. This question is best done without multiplying out the brackets: keep the factors of $\left(\frac{1}{3}x + 2\right)$ intact.
(a) Factorise.
(b) Use the chain rule.
(c) The curve is a positive cubic. Together with the answers in (a) and (b), this is enough to sketch the behaviour.

2067. Position the diagonals of the parallelogram on the x and y axes, and consider the locations of the vertices.

2068. Assume, for a contradiction, that $4p^2 + 1$ is the square of an odd number of the form $2k + 1$. (It couldn't be the square of an even number.)

2069. You can do this by brute force, since there are only 16 outcomes in total. List the successful ones.

2070. The boundary equations are $y = 2 - 3\,|x - 2|$ and $y = x^2$.

2071. In each case, sketch the relevant triangle in 2D, and use trigonometry.

2072. Use the chain rule in the form $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$.

2073. Write the sum out longhand, using the definition of cosec as the reciprocal of sin. Then quote the exact values.

2074. The probability of each colour showing on a single die is $\frac{1}{3}$. So, the possibility space consists of the following nine equally likely outcomes:

	W	B	R
W			
B			
R			

2075. Find the roots. Then differentiate. You will find that the gradient is undefined at one of the roots. This isn't a problem; it means the tangent is in the y direction.

2076. Don't multiply out the brackets. Rather, factorise the difference of two squares, and then take out common factors.

2077. This is an exercise in visualising outcomes. Colour one face, and then use the diagram to establish how the other faces must be coloured, if no two coloured faces are to share an edge. It's easier to use a combinatorial approach (counting outcomes) rather than a conditioning approach (multiplying probabilities).

2078. The problem is the meaning of $\frac{dy}{dx}$: the gradient m should be a constant.

2079. Since $y = x$ is a diameter of the circle $x^2 + y^2 = 4$, the region required is a semicircle.

2080. Differentiate both sides with respect to u, then use the fact that $\frac{dt}{du}$ and $\frac{du}{dt}$ are reciprocals.

2081. Assume the square has unit side length with the origin at the bottom left, and call the coordinates of the point inside it (x, y). Then find the area of the two unshaded triangles; y should cancel.

2082. Use implicit differentiation, i.e. the chain rule.

2083. (a) The variance is given by $s_x^2 = \frac{1}{n}(\Sigma x_i^2 - n\bar{x}^2)$. Rearrange this to give Σx_i^2 as the subject, and substitute into an expression for $\bar{y}$.
   (b) The formula for s_y requires Σy_i^2. Explain why this is unknown.

2084. Start with the following:

$$\lim_{h\to 0} \frac{a\,\text{f}(x+h) + b\,\text{g}(x+h) - (a\,\text{f}(x) + b\,\text{g}(x))}{h}$$
$$= \lim_{h\to 0} a\frac{\text{f}(x+h) - \text{f}(x)}{h} + ...$$

2085. Write the sum longhand, and solve.

2086. Solve for the intersections. Then set up a definite integral, cumulating the difference in y values.

2087. This is, in a sense, somewhat obvious. Results that are somewhat obvious tend to be most easily proved by a thoroughly obvious contradiction. So, assume that there is a smallest positive rational number $\frac{p}{q}$. Then construct a smaller one!

2088. The right-hand factor is a quadratic in 3^x. Be careful to consider the appropriate domains: an exponential can only give a positive output, or, equivalently, logarithms only take positive inputs.

2089. (a) Set the first derivative to zero.
   (b) There are three roots with two SPs between them. So, the central root must be between the SPs found in (a). Find the central root, and then use the factor theorem to find k.

2090. In $[0, 2\pi)$, consider the number of roots of each equation.

2091. Find out how many terms have three digits by equating the nth term formula to 100. Also verify that the 40th term has three digits.

2092. Consider the equation as a quadratic in a, with coefficients in terms of b. Solve the equation using the quadratic formula.

2093. (a) Set the contact force on the rod at the hinge up as horizontal and vertical components. Having done so, there should be four forces.
   (b) Use horizontal and vertical equilibrium and moments about the hinge.

2094. This statement does not hold. The error is in the equals sign.

2095. The first and third are true, the second is not.

2096. Differentiate by the chain rule, using the standard derivatives for e^x and $\sin x$.

2097. Both statements are true when $x = y$. Consider whether $x = -y$ makes both statements true.

2098. Find the intersections (in terms of k), and set up a single definite integral. Equate this to $\frac{8}{3}$, carry out the integral and solve.

2099. (a) 219 is divisible by 3.
   (b) Multiply out using the binomial expansion, and simplify. Then equate coefficients of $\sqrt{2}$ and $\sqrt{3}$. Divide by 3 in the latter equation, and then use the fact that 73 is prime.

2100. (a) Differentiate.
   (b) The Newton-Raphson iteration is

$$x_{n+1} = x_n - \frac{\text{f}(x_n)}{\text{f}'(x_n)}.$$

   Almost any starting point will find the root.
   (c) Use the factor theorem. If $x = \frac{p}{q}$ is a rational root, then $(qx - p)$ is a factor.

——— END OF 21ST HUNDRED ———